

# First-Order DC Electric Motor Model

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## Nomenclature

|                    |                 |               |                     |
|--------------------|-----------------|---------------|---------------------|
| $\Omega$           | rotation rate   | $v$           | terminal voltage    |
| $Q_m$              | torque          | $\mathcal{R}$ | resistance          |
| $P_{\text{shaft}}$ | shaft power     | $i$           | current             |
| $\eta_m$           | efficiency      | $i_o$         | zero-torque current |
| $K_V$              | speed constant  |               |                     |
| $K_Q$              | torque constant |               |                     |

## 1 Motor Model

### 1.1 Fundamental relations

The behavior of DC electric motor is described by the equivalent circuit model shown in Figure 1.

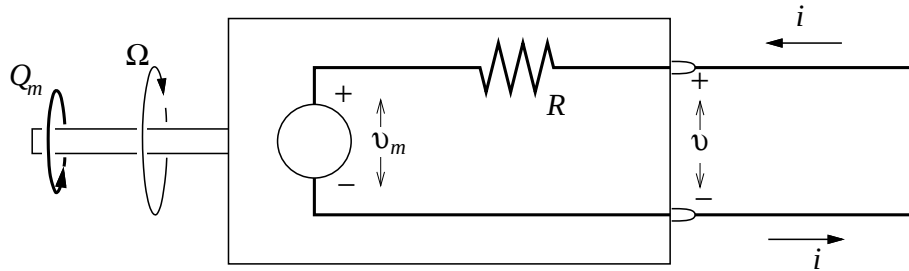


Figure 1: Equivalent circuit for a DC electric motor.

#### 1.1.1 Resistance model

The resistance  $\mathcal{R}$  of the motor is assumed to be constant.

#### 1.1.2 Torque model

The shaft torque  $Q$  is assumed proportional to the current  $i$  via the torque constant  $K_Q$ , minus a friction-related current  $i_o$ .

$$Q_m(i) = (i - i_o)/K_Q \quad (1)$$

#### 1.1.3 Voltage model

The internal back-EMF  $v_m$  is assumed proportional to the rotation rate  $\Omega$  via the motor speed constant  $K_V$ .

$$v_m(\Omega) = \Omega/K_V \quad (2)$$

The motor terminal voltage is then obtained by adding on the resistive voltage drop.

$$v(i, \Omega) = v_m(\Omega) + i \mathcal{R} = \Omega/K_v + i \mathcal{R} \quad (3)$$

## 1.2 Derived relations

The model equations above are now manipulated to give the current, torque, power, and efficiency, all as functions of the motor speed and terminal voltage. First, equation (3) is used to obtain the current function.

$$i(\Omega, v) = \left( v - \frac{\Omega}{K_v} \right) \frac{1}{\mathcal{R}} \quad (4)$$

The remaining motor variables follow immediately.

$$Q_m(\Omega, v) = \left[ i(\Omega, v) - i_o \right] \frac{1}{K_Q} = \left[ \left( v - \frac{\Omega}{K_v} \right) \frac{1}{\mathcal{R}} - i_o \right] \frac{1}{K_Q} \quad (5)$$

$$P_{\text{shaft}}(\Omega, v) = Q_m \Omega \quad (6)$$

$$\eta_m(\Omega, v) = \frac{P_{\text{shaft}}}{i v} = \left( 1 - \frac{i_o}{i} \right) \frac{K_v}{K_Q} \frac{1}{1 + i \mathcal{R} K_v / \Omega} \quad (7)$$

In the limiting case of zero friction ( $i_o = 0$ ) and zero resistive losses ( $\mathcal{R} = 0$ ), the efficiency (7) becomes

$$\eta_m = \frac{K_v}{K_Q} \quad (\text{zero losses}) \quad (8)$$

Hence, energy conservation requires that the torque constant  $K_Q$  must be equal to the speed constant  $K_v$ . The equations here assume  $K_v$  is in rad/s/Volt, and  $K_Q$  is in the equivalent units of Amp/Nm. However,  $K_v$  is usually given in RPM/Volt.

## 2 Motor Parameter Measurement

The motor operation functions (4) – (7) depend on the “motor constants”  $\mathcal{R}$ ,  $i_o$ ,  $K_v$ ,  $K_Q$ . These can be obtained by benchtop measurements, together with simple data fitting.

### 2.1 Motor resistance

The motor resistance  $\mathcal{R}$  can be measured directly with a milli-ohmmeter. Alternatively, it can be determined using a power supply, an ammeter, and a voltmeter. A representative range of currents  $i$  is sent through the motor by applying a suitable voltages  $v$  to the motor terminals, while the shaft is held to prevent rotation. The resistance is then computed using Ohm’s Law.

$$\mathcal{R} = v/i \quad (9)$$

On a commutated motor this will likely vary with shaft position, in which case the various  $\mathcal{R}$  values need to be averaged over different shaft positions.

## 2.2 Zero-load current

With the motor shaft free to turn, a voltage  $v$  is applied to the motor such that the RPM is comparable to the expected operating RPM. The resulting zero-load current  $i_o$  is then measured.

## 2.3 Speed constant

Using the previously-obtained resistance  $\mathcal{R}$ , the motor back-EMF voltage  $v_m$  can be computed for the freely-spinning case, using the measured zero-load  $v$ ,  $i_o$  data.

$$v_m = v - i_o \mathcal{R} \quad (10)$$

The speed constant is then computed from (2).

$$K_v = \Omega / v_m \quad (11)$$

## 2.4 Torque constant

The torque constant can be simply assumed to be the same as  $K_v$ .

$$K_Q = K_v \quad (12)$$

Alternatively, it can be obtained from motor torque data if this is available. Ideally, the motor is operated at different loads (the zero-load tests are not usable here), and over a range of speeds by applying different voltages. According to the torque model (1), the  $Q_m$  data versus  $i - i_o$  should be a straight line passing through the origin. The slope of the best-fit line to the data then gives  $1/K_Q$ .

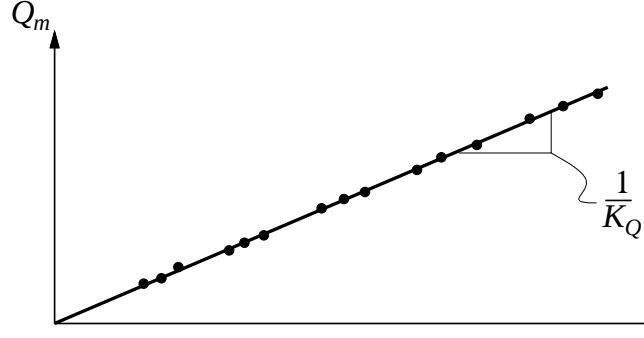


Figure 2:  $Q_m$  data versus  $i - i_o$ , with curve fit to determine  $K_Q$ .

As can be seen from (8), any resulting discrepancy between this measured  $K_Q$  and  $K_v$  will give a nonunity efficiency even if all the loss quantities  $\mathcal{R}$ ,  $i_o$  are set to zero. Hence, a nonunity  $K_v/K_v$  ratio indicates the degree of imperfection of the present motor model. However, simply allowing  $K_Q$  to be different from  $K_v$  will at least partially account for the model imperfections, since the efficiency is then likely to be predicted more accurately.